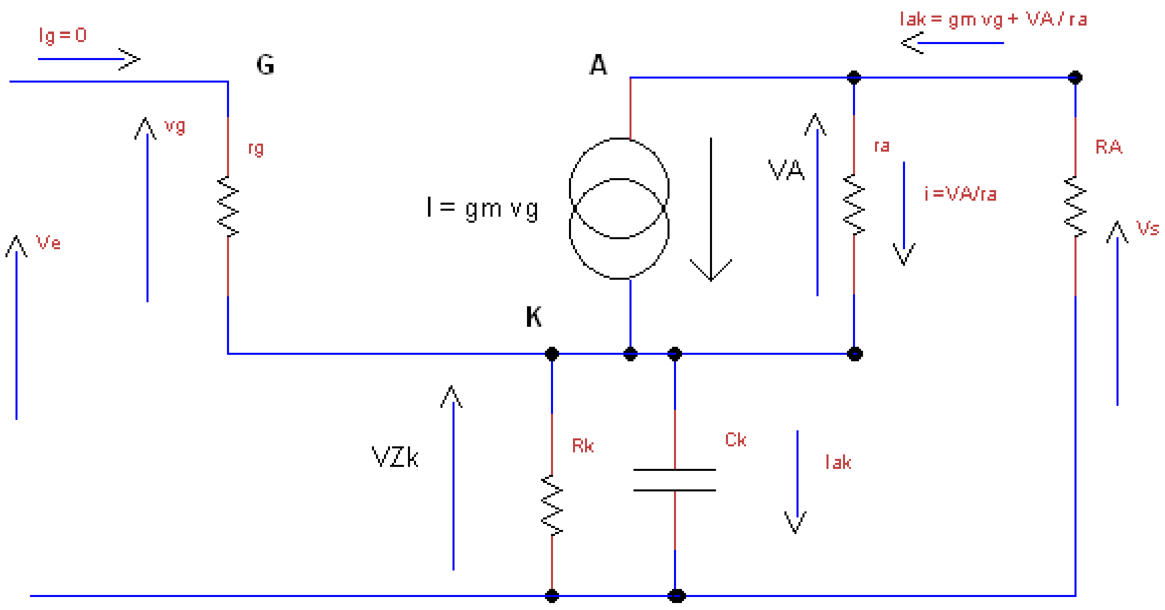
Common cathode triode amplifier
Low cut-off frequency

common cathode amp stage with partially bypassed cathode



With :

- ra = internal resistance of the triode (cf Datasheet)
- VA = anode – cathode voltage
- gm = transconductance (cf Datasheet)
- Zk = complex impedance on the cathode (Rk // Ck – with Ck being a NOT perfect capacitance)
- RA load impedance on the anode
- rg = grid resistance held as infinite

The gain of the stage is :

$$\boxed{Av = \frac{Vs}{Ve} = \frac{-\mu\, RA}{RA + ra + (\mu + 1)Zk}} \qquad (1)$$

This formula helps computing precisely the low cut-off frequency induced by the bypass capacitor Ck

In the bandwidth Zk = 0 and the gain is $Av = \frac{-\mu RA}{RA + ra}$.

In fact **Zk = Rk // Ck =** $\frac{Rk}{1 + jCkRk \ \omega}$

We see that for $\omega = \infty$ Zk = 0 and the gain is maximum. The cut-off frequency is confirmed as a low frequency and the problem becomes how low the value of ω_0 can be before the gain reaches the value of **Av – 3 dB** ?

In fact for a same value of Ve (Vin) we will reach tle low limit of the bandwidth when Vs (Vout) will be diminished by 3 dB , therefore at the cut-off $Vsc = \frac{Vs}{\sqrt{2}}$.

For this it is sufficient that the magnitude of the denominator of (1) equals $\sqrt{2}(RA + ra)$.

At the cut-off frequency ω_0 we have :

$$\sqrt{2}\,(RA + ra) = \left| RA + ra + (\mu + 1)\frac{Rk}{1 + jCkRk \ \omega_0} \right|$$

$$\sqrt{2} = \frac{\left| RA + ra + (\mu + 1)\frac{Rk}{1 + jCkRk \ \omega_0} \right|}{\left| RA + ra \right|}$$

multiplied high and low by **1 + jCkRkw0** it yields :

$$\sqrt{2} = \frac{\left| (RA + ra)(1 + jCkRk \ \omega_0) + (\mu + 1)Rk \right|}{\left| (Ra + ra)(1 + jCkRk \ \omega_0) \right|}$$

$$\sqrt{2} = \frac{\left| (RA + ra) + (\mu + 1)Rk + j(RA + ra)RkCk \ \omega_0 \right|}{\left| RA + ra + j(RA + ra)RkCk \ \omega_0 \right|}$$

Hence simplified by Ra+ra :

$$\sqrt{2} = \frac{\left| 1 + \frac{(\mu + 1)Rk}{Ra + ra} + jRkCk \quad \omega_0 \right|}{\left| 1 + jRkCk \quad \omega_0 \right|}$$

Dividing the magnitudes yields :

$$\sqrt{2} = \frac{\sqrt{(1 + \frac{(\mu + 1)Rk}{Ra + ra)})^2 + Rk^2 Ck^2 \omega_0^2}}{\sqrt{1 + Rk^2 Ck^2 \omega_0^2}}$$

Squaring it all :

$$2 + 2\,Rk^2 Ck^2 \omega_0^2 = (1 + \frac{(\mu + 1)Rk}{Ra + ra)})^2 + Rk^2 Ck^2 \omega_0^2$$

$$2 + Rk^2 Ck^2 \omega_0^2 = (1 + \frac{(\mu + 1)Rk}{Ra + ra)})^2$$

$$\omega_0^2 = \frac{(1 + \frac{(\mu + 1)Rk}{Ra + ra)})^2 - 2}{Rk^2 Ck^2}$$

With $\omega_0 = 2\pi\ f$ (Hz)

$$\boxed{f_{(Hz)} = \frac{1}{2\pi RkCk} \sqrt{\left[1 + \frac{(\mu + 1)Rk}{RA + ra}\right]^2 - 2}}$$

Of course if f is given , the value of the capacitance Ck can be found with the same formula, just reversing both.